# By Definition

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Most activist groups that go against the status quo have something they say, that just instills oneself with that cringe feeling all over your body. Creationists have the question "If we evolved from monkeys, then why are there still monkeys?". Flat Earthers have "Water always finds its level!"...and New Atheists™ have "by definition". Whenever I hear a New Atheist™ say those two concatenated words, a reflexive visceral reaction occurs inside me, that I feel deep inside my bones, and I brace myself for the almost inevitable onslaught of linguistic misunderstandings, followed by assertions of an improper use of a descriptive dictionary.

More specifically, the phrase that New Atheists™ frequently say to me is "Atheism is, by definition...". When uttering this phrase, regardless of what follows thereafter, they are making a fundamental mistake of trying to coax a prescription out of a description. It has been well established, contrary to assertions made by fiat of groups such as American Atheist, that words like "atheism" are polysemous and do not have a singular definition which is both necessary and sufficient for a prescribed usage. Typically, prescribed definitions are generally found in fields like math and logic. For example, the relationship of 0!=1 is true because there is a specific definition of the word "factorial" that exists, that has both necessary and sufficient conditions to establish that relationship as true.

In this case, it is first established by the definition of a "factorial" from which we can expand upon our definition of factorial for a special case, and say $0! = 1$, *by definition*:

(from Wolfram MathWorld)

$n! \equiv n(n-1)\cdots 2 \cdot 1$ where n is a positive integer, or where n $\geq 1$ as n $\in \mathbb{N}$

which can be more generally expanded to n! $= n(n-1)\cdot(n-2)\cdot(n-3)\cdots 3\cdot 2\cdot 1$. Notice in the definition that a triple bar is used to represent the relationship is stronger than a mere equality, and as such, it is giving a prescriptive definition. A prescriptive definition tells you that in a math proof, you could literally give justification of an the expansion above to be replaced by n!, *by definition*.

Examples of factorials using the definition of a factorial:[1]

$5! = 5(5-1)\cdot(5-2)\cdot 2\cdot 1 = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$
$4! = 4(4-1)\cdot 2\cdot 1 = 4\cdot 3\cdot 2\cdot 1 = 24$
$3! = 3(3-1)\cdot 1 = 3\cdot 2\cdot 1 = 6$
$2! = 2(2-1) = 2$
$1! = 1$

We can also express the definition of factorial a bit more mathematically as:

$$n! = \prod_{i=1}^{n} i$$

Where "Π" is capital pi to mean a summation of products similar to how Σ is a summation for addition from 1 to n (the index) with the argument i. So, if n 5 then:

$$5! = \prod_{i=1}^{5} i = 120$$

This also gives up a nice recurrence relation of:

$n! = n(n-1)!$

[1]Each time we have a decrease in n, we have to remove one of the quantities being multiplied, so 5! has 5 things being multiplied, 4! has 4 things being multiplied, 3! has 3 things being multiplied, 2! has 2 things being multiplied, 1! has only 1 element so nothing is being multiplied.

Examples using recurrence relation:

$10! = 10 \cdot 9!$
$42! = 42 \cdot 41!$
$99! = 99 \cdot 98!$

If you have $n = 1$ then given that $n! = n(n-1)!$ then:

$1! = 1 \cdot 0!$

For that to work then clearly 0! must equal 1 to adhere to our initial definition as $1! = 1 \cdot 1! = 1$. However, if you notice by the definition of a factorial n is only for positive numbers. 0 is not a positive number, so we have to specially define 0! to make it work with our definition of factorials, as even with our capital pi notation since the index has to start at 1 which is the first positive number.

So we can expand upon our definition of factorial for a special case, and say $0! = 1$, *by definition*, as the product of an empty set is 1 by the "empty product rule". Which means if you take the product of a set with no elements (empty set or $\varnothing$), it equals 1 by the multiplicative identity which is the product equivalent of the additive identity of zero in addition. If you add up no numbers, like adding the elements for an empty set $\varnothing$, it is equal to the addition identity of 0, by definition. If you multiply "the elements" in an empty set (one that contains no elements), it is equal to the multiplicative identity of 1, *by definition.*

There are other mathematical examples of "by definition" which I won't go into here, but suffice it to say that it is based upon ring theory. It should be intuitive enough to see that you can't actually add elements which are not there so, we have to find a different way to say adding of zero elements is zero (addition identity). We do the same for products by saying the multiplication of zero elements, product of an empty set $\varnothing$, is by definition 1 (multiplicative identity).

This also comes from combinatorics as to how many ways can we arrange (permutations) a set that contains zero elements, or more intuitively, how many ways can we do nothing. One way.

So we can say the definition of 0! is 1 and when we see 0! replace it with 1. For example in a Taylor series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

This expands out to:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

In the first term in the expansion 1/0! the expansion would only hold if of course $0! = 1$ to give us $1/0! = 1/1 = 1$ which is why you often see Taylor expansions merely written as $1 + x + x^2/2! + x^3/3!$ Since we defined 0! to be 1 and by our definition of factorials $1! = 1$ If you don't believe me, just try doing a Taylor series where $0! \neq 1$ and let me know how well that works for you.

You may be wondering what all this has to do with New Atheists™ saying "Atheism, by definition...", but this above is an example of a prescriptive definition that is given by the symbol "0!" so we can, *by definition*, replace it with 1 in our mathematical computations. More importantly, as far as the conceptual take away, 0! is not "describing" the number 1, it is by definition equal to the number 1.

A dictionary however does not prescribe usages, it merely describes synchronic usage of modern language and its usage in the population. It is usually in the form of:

$\langle object \rangle$ = general description of the object

Contrary to a descriptive definitional form, the form for a prescriptive definition is more akin to:

$\langle object \rangle \equiv$ substitutional equivalence from necessary and sufficient conditions to establish the relationship.

A popular reference for New Atheist™ to cite as a definition of an atheist is:

Atheist = "a person who disbelieves or lacks belief in the existence of God or gods."

This merely is given a description of an atheist, it is not prescribing that anyone who lacks a belief is "by definition" is an atheist. Merely, that 1) "lack of belief" describes a person who is an atheist. 2) as their definition of atheist as to what they believe constitutes being an atheist.

A mathematical example would be equivalent to:

Square = A four sided object.

This is a descriptive definition. It describes, albeit not very well, what a square is...it is very general, but not untrue.

We can have other more specific definitions such as:

Square = A two dimensional plane object consisting of four congruent sides and four right 90° angles

That certainly is a more precise description of a square and sets it up as a special case parallelogram, kite, quadrilateral, rectangle, rhombus, and trapezoid...but it is still merely describing what the object we call a square has for attributes. It meets both necessary and sufficient conditions to not just describe a square, but that if you had an object that met all those conditions (being two dimensional, in a plane, having four sides of equal length, and four 90° angles), you could call it a square. Unlike our first definition of atheist, which was just a general definition which contained a necessary condition for to be a square (having four sides), but obviously was quite insufficient to be a prescriptive definition.

So when a New Atheist™ says "Atheist, by definition, is a person who lacks a belief", they are erroneously attempting to make a description that is not prescriptive, into a prescriptive definition either by ignorance or by deceit. All atheists lack a belief God exists, and the definition is a true "description" of an atheist. It, however, is not prescribing that all who lack a belief *are* atheists.

Equivalently, a descriptive definition for theist would be:

Theist = "a person who disbelieves or lacks belief in the non-existence of God or gods"

That definition accurately describes all theists, as all theists disbelieve, which entails lack of belief in the non-existence of God. The reason dictionaries don't have that specific definition, is that theists don't use "lack of belief in the non-existence of God or gods" in their usage of the word theist, else the dictionary would reflect that particular usage.

If you therefore allow for the word "atheist" to be as "a person who disbelieves or lacks belief in the existence of God or gods" to be a prescriptive definition, derived from a descriptive one, then you must also allow theist to have the equivalent of "theist" to be prescriptively define as a person who disbelieves or lacks belief in the non-existence of God or gods." to be prescriptive as well, else you're guilty of special pleading. (See my WASP argument)

Which additionally means anyone who lacks a belief in the existence and non-existence of God, could be both an "atheist" and a"theist" at the same time due to the how the definitions are prescribed. (See my Atheist Semantic Collapse argument).

See how these things I discuss in my blog, and on my channel, all tie in together and just start to fall apart when imprecise usages of terms are prescribed? So next time a New Atheist™ says "Atheism, by definition is…", see if they understand the difference between a descriptive definition and a prescriptive one. I personally can't recall ever asking one who was able to tell me the difference, but if you do ever find one...I would love to know about it!

—Steve McRae